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# Improved Schwinger-Dyson approach to pairing phenomena and QCD phase diagram*)

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**Abstract**

Phase structure and phase transitions in dense QCD are studied using the Cornwall-Jackiw-Tomboulis (CJT) potential in the improved ladder approximation. The gap function, the condensation energy and the structure of Cooper pairs are investigated at finite temperature and density. Due to strong coupling effects at low densities, the gap, critical temperature and their ratio deviate from the weak coupling values. It is shown that the internal structure of Cooper pairs are robust against the thermal effects, despite the fact that the pairs are strongly correlated near the critical temperature. Also, the effect of the strange quark mass, $m_s$ on the phase diagram is examined using a simple kinematical criterion. We discuss the behavior of the unlocking line, on which the CFL turns into the 2SC, through the variation of $m_s$.

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## §1. Introduction

The color superconducting phases at high baryon density have attracted much interest in high density QCD. In particular, the two types of the pairing pattern, namely color-flavor locked (CFL) pairing and iso-scalar (2SC) pairing have been studied by many authors and rich physics in these phases has been revealed so far.[1)]

In the chiral limit, weak coupling analyses of the gap equations reveal the CFL dominance over the 2SC at zero temperature and the coincidence of critical temperatures of transitions to the QGP phase from the 2SC state and from the CFL state. The former is attributed to a competition between the number of degrees of freedom participating in the pairing correlation and the sizes of the gaps. The latter is due to two facts: The rapid vanishing of the pairing in the color sextet channel as the critical temperature $T_c$ is approached, and the disappearance of the nonlinearity of the gap equation in the diquark condensate near $T_c$. However, in the low density regime, the sizes of gaps are modified from the weak coupling values owing to various strong coupling effects.[2)] Also, the pairing in the color symmetric channel may become relevant at low densities, although it is suppressed by one coupling constant relative to the anti-triplet channel in the weak coupling regime.

We examine in this article the Schwinger-Dyson (SD) approach[3)] in the improved ladder approximation to the pairing phenomena at finite temperature and density. This approach, the SD equation in the Landau gauge with an improved running coupling constant, can reproduce the reasonable vacuum physics related to the chiral symmetry breaking as well as the perturbative result for the gap in the weak coupling regime. Various strong coupling effects manifest themselves for realistic densities in this approach.[2)] More complete and detailed analysis was done in Ref.[4)]

## §2. Gaps, correlation and thermodynamic grand potential

The gap equation is nothing but the self-consistency condition for the proper self energy $\hat{\Delta}_{\pm}(p_0, p)$ in the Nambu-Gor'kov base $\Psi = (q, i\gamma^2\gamma^0\bar{q}^t)$

$$\boldsymbol{\Sigma}(p_0, \boldsymbol{p}) = ig^2 \int \frac{d^4q}{(2\pi)^4} \boldsymbol{\Gamma}^{\mu}_{a}\, \boldsymbol{S}(q_0, \boldsymbol{q}) \boldsymbol{\Gamma}^{\nu}_{b}\, D^{ab}_{\mu\nu}(q_0 - p_0, \boldsymbol{q} - \boldsymbol{p}). \tag{2·1}$$

Here, $D^{ab}_{\mu\nu}$ is the in-medium gluon propagator, $\boldsymbol{S}$ is the full quark propagator, and $\boldsymbol{\Gamma}^{a}_{\mu} = \gamma_{\mu}\lambda^a/2$ is the quark-gluon vertex. The gap matrix is defined by the off-diagonal component of the self energy $\boldsymbol{\Sigma}$. The 2SC ansatz for the gap matrix is

$$\hat{\Delta}^{\rm 2SC}_{\pm}(p_0, \boldsymbol{p}) = (\tau_2 \times \lambda_2)^{ab}_{ij} \Delta_{\pm}(p_0, \boldsymbol{p}). \tag{2·2}$$

$\Delta_{+(-)}$ is the positive (negative) energy component of the gap matrix $\Delta(p_0, \boldsymbol{p}) = (\Delta_- + \Delta_+) - (\Delta_+ - \Delta_-)\gamma^0 \boldsymbol{\gamma} \cdot \hat{p}$. The color-flavor locked gap matrix is

$$\hat{\Delta}^{\mathrm{CFL}}_{\pm}(p_0, p) = \frac{1}{3}\delta^a_i \delta^b_j \Delta^1_{\pm}(p_0, \boldsymbol{p}) + \left(\delta^a_j \delta^b_i - \frac{1}{3}\delta^a_i \delta^b_j\right) \Delta^8_{\pm}(p_0, \boldsymbol{p}). \tag{2·3}$$

Index $(a, b, \cdots)$ and $(i, j, \cdots)$ represents color and flavor, respectively. We use the quasi-static approximation of the hard dense loop propagator in the Landau gauge.[2),4)] Following Ref.,[2)] we replace the coupling $g^2$ with the momentum dependent effective coupling $\bar{g}^2(p, q)$ using the Higashijima-Miransky prescription.

$$\bar{g}^2(p, q) = \frac{16\pi^2}{\beta_0} \frac{1}{\ln((p^2_{\max} + p^2_c)/\Lambda^2)}, \quad p_{\max} = \max(p, q), \tag{2·4}$$

where $\beta_0 = (11N_c - 2N_f)/3$, and $p^2_c$ plays the role of an infrared regulator. We adopted $\Lambda =$ 400 MeV and $p^2_c = 1.5\Lambda^2$ for our numerical calculations.The correlation function describes the off-diagonal long range order in the system:

$$\phi_+(q) = \tanh\left(\sqrt{(q-\mu)^2 + \Delta^2_+(q)}\Big/2T\right) \frac{\Delta_+(q)}{2\sqrt{(q-\mu)^2 + \Delta^2_+(q)}}. \tag{2·5}$$

$\Delta(q)$ is the gap for the quasi-quark having momentum $q = |\boldsymbol{q}|$. $\phi_+(p)$ are defined by $\langle 2\mathrm{SC}| a^a_i(p) a^b_j(-p) |2\mathrm{SC}\rangle = (\tau_2\lambda_2)^{ab}_{ij}\phi_+(q)$, where $a$ is the annihilation operator for quark. We have omitted the helicity indices here. We can calculate the coherence length from the correlation function which characterizes the size of the quark Cooper pair.[2),4)] Using the method developed by Cornwall, Jackiw and Tomboulis,[5)] the thermodynamic potential par unit volume relative to the normal Fermi gas up to 2-loop order is given by $\delta\Omega(\mu, T) = \frac{T}{2V}\left\{\mathrm{TrLog}\left[\boldsymbol{S}\boldsymbol{S}_0^{-1}\right] - \frac{1}{2}\mathrm{Tr}\left[\boldsymbol{S}\boldsymbol{\Sigma}\right]\right\}$. $\boldsymbol{S}_0$ is bare quark propagator in the Fermi sea. Note that this expression is only valid at the stationary point determined by solving the gap equation for $\boldsymbol{\Sigma}$.

## §3. Numerical results and discussion

We present the numerical solutions for the gap equations at $\mu = 1000$ MeV corresponding to about 80 times the normal nuclear saturation density $\rho_0 = 0.17$ fm$^{-3}$. We display the gap function evaluated on the quasi-quark mass shell $\Delta_{\pm}(p)$ for the 2SC state Fig. 1 (a), and the singlet and octet gap functions, $\Delta_{1\pm}(p)$ and $\Delta_{8\pm}(p)$, for the CFL state (b). The shapes of these gap functions are all similar as functions of momentum, except for their magnitudes $|\Delta_8(p)| < |\Delta(p)| < |\Delta_1(p)|$. In order to see the characteristics of the phase transitions in detail, we show the temperature dependence of the gaps at the Fermi surface Fig. 1 (c) and that of the CJT condensation energies for the 2SC and CFL states (d). The phase transitions

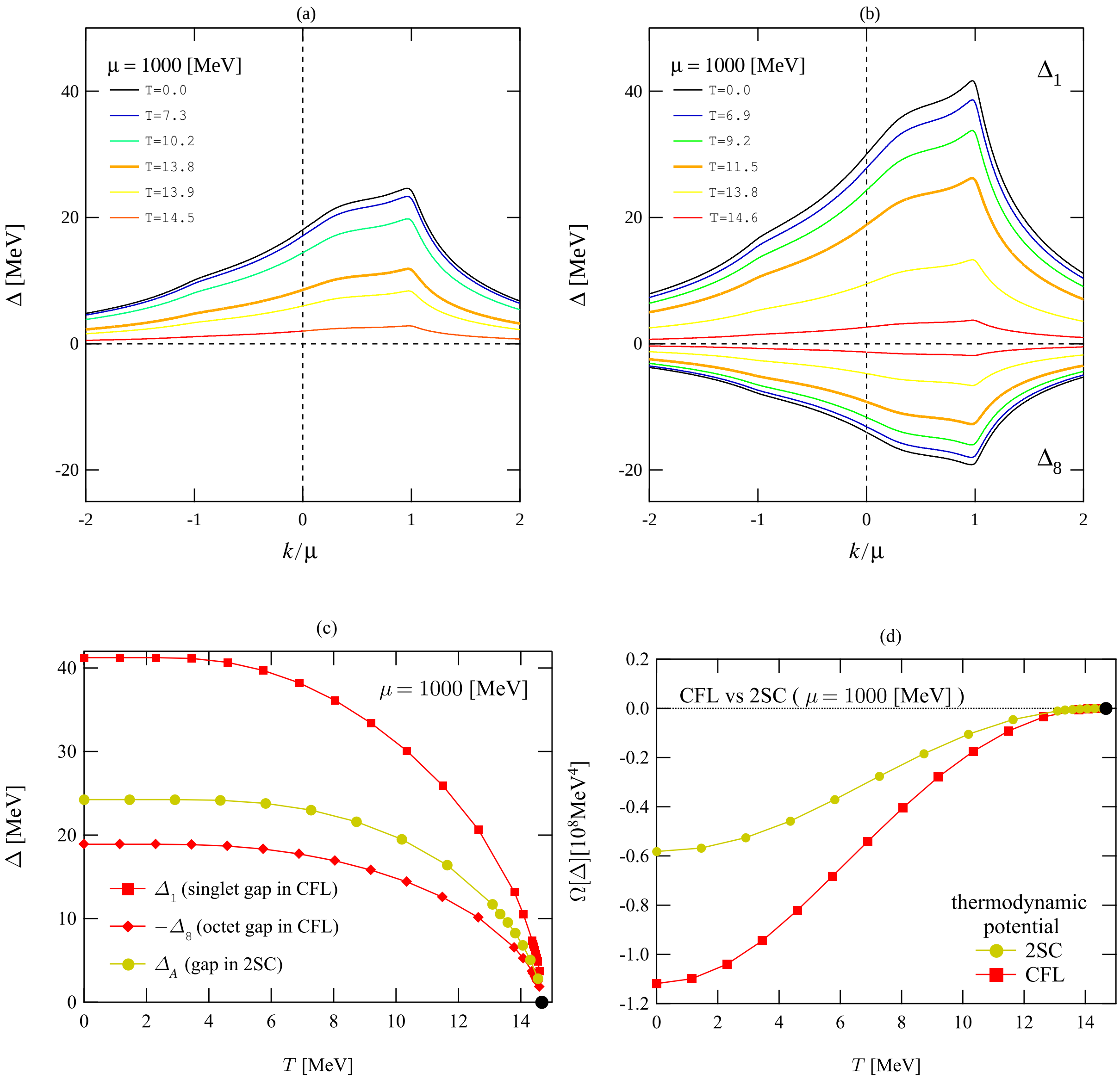


Fig. 1. (a) Gap functions at $\mu = 1000\mathrm{MeV}$ for the 2SC state. The values $(-|k|, \Delta_-(|k|))$ are plotted in left half of the figure. (b) Gap functions for the CFL state. (c) The temperature dependence of the gaps at the Fermi surface. (d) The condensation energy as function of temperature.

to the QGP phase both from the 2SC state and from the CFL phase are of 2nd order, as in the BCS theory with a contact interaction: The derivatives of the gaps seem to diverge as $T_c$ is approached in (c), and the thermodynamic bulk quantities in the superconducting phases connect continuously to those of the normal QGP phase (d). The size of Cooper pair $\xi_c$, the root mean square radius of the Cooper pair with wave function $\phi_+$ is displayed in Fig. 2 (a). $\xi_c$ is not affected significantly by changes in the temperature. This shows the fact that the

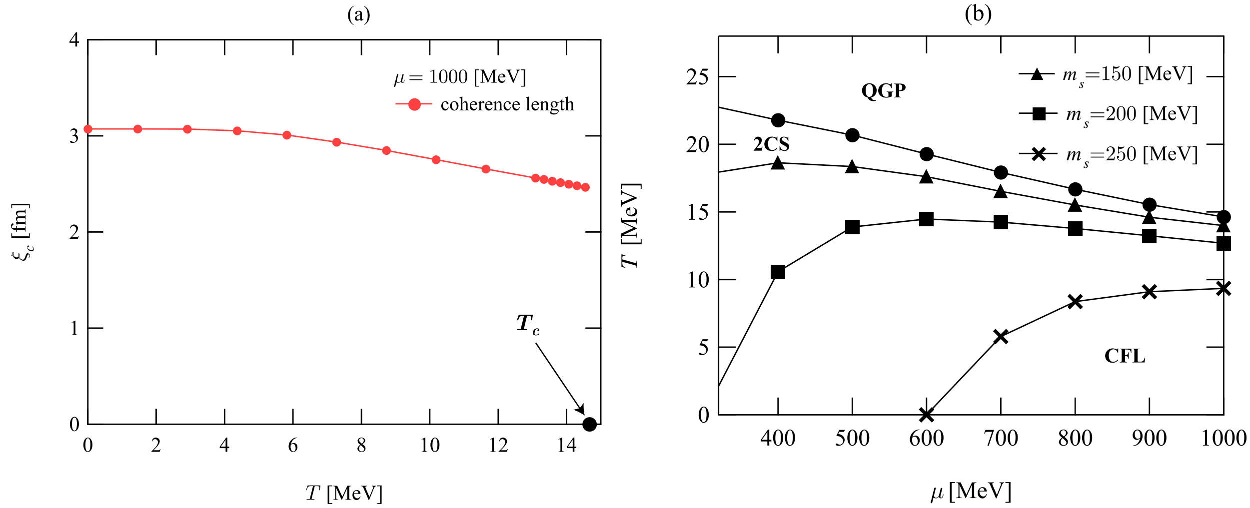


Fig. 2. (a) The size of 2SC Cooper pair as a function of temperature at $\mu = 1000$MeV. (b) The calculated QCD phase diagram from the improved Schwinger-Dyson approach.

Cooper pairs are robust against the thermal fluctuation, although the correlation between diquarks diverge as the critical temperature is approached because $\Delta(q) \to 0$ as $T \to T_c - 0$. This fact also suggests the existence of the incoherent Bose gas of tightly bound diquarks just above $T_c$.

Fig. 2 (b) shows our result for the QCD phase diagrams for $m_s = 0,\ 150,\ 200$ and 250 MeV. In the chiral limit, there is only one line (black bold dots) dividing the $(\mu, T)$ plane into the QGP and CFL phases by 2nd order transition. The critical temperature is increased as one goes towards low density. This is due to the strong coupling effects including participation of antiquarks to pairing correlation. It is surprising that the critical temperatures of transitions to the QGP phase from the 2SC and CFL states exactly coincides. The unlocking critical line on which the CFL state turns into 2SC state by 1st order transition appears for finite value of $m_s$, and shifts from the triangles to the squares, and to crosses, with the increase of $m_s$. These were estimated using a kinematical criterion for the unlocking.[6),7)] The strength of the 1st order transition becomes weaker as the quark density is increased, and the critical end point at which the 1st order transition terminates is located at $\mu = \infty$. If the strange quark mass is lower than 200 MeV, then chirally broken vacuum phase might be continuously connected to the CFL phase without phase transition.

## References

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